

# Superluminal localized solutions to Maxwell equations propagating along a normal-sized waveguide

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We show that localized (nonvanishing) solutions to Maxwell equations exist, which propagate without distortion along normal waveguides with superluminal speed.

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## I. INTRODUCTION: LOCALIZED SOLUTIONS TO THE WAVE EQUATIONS

Already in 1915 Bateman [1] showed that Maxwell equations admit (besides of the ordinary solutions, endowed in vacuum with speed  $c$ ) of wavelet-type solutions, endowed in vacuum with group velocities  $0 \leq v \leq c$ . But Bateman's work went practically unnoticed. Only few authors, as Barut and co-workers [2], followed such a research line; incidentally, Barut and coworkers constructed even a wavelet-type solution traveling with superluminal group velocity [3]  $v > c$ .

In recent times, however, many authors have discussed the fact that all (homogeneous) wave equations admit solutions with  $0 < v < \infty$ : see, e.g., Donnelly and co-workers [4]. Most of these authors confined themselves to investigate (subluminal or superluminal) *localized* nondispersive solutions in vacuum: namely, those solutions that were called “undistorted progressive waves” by Courant and Hilbert. Among localized solutions, the most interesting appeared to be the so-called “X-shaped” waves, which—predicted to exist even by special relativity in its extended version [5]—had been mathematically constructed by Lu and Greenleaf [6] for acoustic waves, and by Ziolkowski *et al.* [7], and later Recami [8], for electromagnetism.

Let us recall that such “X-shaped” localized solutions are superluminal (i.e., travel with  $v > c$  in the vacuum) in the electromagnetic case; and are “supersonic” (i.e., travel with a speed larger than the sound speed in the medium) in the acoustic case. The first authors to produce X-shaped waves *experimentally* were Lu and Greenleaf [9] for acoustics, Saari and co-workers [10] for optics, and Mugnai and co-workers and for microwaves.

## II. ABOUT EVANESCENT WAVES

Notwithstanding all that work, still it is not yet well understood what solutions (let us now confine ourselves to Maxwell equations and to electromagnetic waves) have to enter into the play in some experiments. Actually, most of the experimental results did not refer to the above-mentioned localized, subluminal or superluminal, solutions, which in vacuum are expected to propagate rigidly (or almost rigidly, when suitably truncated). They referred, on the contrary, to measurements of the group velocity of *evanescent waves* (cf., e.g., Refs. [11,12]). In fact, tunneling wave packets (tunneling photons too) and/or evanescent waves had been predicted to be superluminal by *both* quantum mechanics [13] and special relativity [5].

For instance, experiments [12] with evanescent waves traveling down an undersized *waveguide* revealed that evanescent modes are endowed with superluminal group velocities [14].

A *problem* arises in connection with experiments with two “barriers” (i.e., segments of *undersized* waveguide) 1 and 2 separated by a *normal-sized* waveguide 3. In fact, it was found that for suitable frequency bands the wave coming out from barrier 1 goes on having a practically infinite speed, and crosses the intermediate (normal) waveguide 3 in zero time [15]. Even if this can be theoretically understood by looking at the relevant transfer function (see the computer simulations, based on Maxwell equations only, in Refs. [14,16,17]), it is natural to ask ourselves whether solutions to the Maxwell equations can actually exist, that travel with superluminal speed in a normal waveguide (where one ordinarily meets propagating, subluminal modes only).

Namely, the dispersion relation in undersized guides is  $\omega^2/c^2 - k^2 = -\Omega^2$ , so that the standard formula  $v \approx d\omega/dk$  yields a  $v > c$  group velocity [17,18]. However, in normal guides the dispersion relation becomes  $\omega^2/c^2 - k^2 = +\Omega^2$ , so that the same formula yields values  $v < c$  only.

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In this paper we are going to show that actually localized solutions to Maxwell equations propagating with  $v > c$  do exist even in normal waveguides; but their group velocity  $v$  cannot be given<sup>1</sup> by the approximate formula  $v \approx d\omega/dk$ . One of the motivations of the present paper is just contributing to the clarification of this question, even if our localized superluminal solutions are *not* strictly related with the particular case in Ref. [15].

### III. ABOUT SOME LOCALIZED SOLUTIONS TO MAXWELL EQUATIONS

Let us start by considering localized solutions to Maxwell equations in vacuum. A *theorem* by Lu *et al.* [19] showed how to start from a solution holding in the *plane*  $(x, y)$  for constructing a three-dimensional solution rigidly moving along the  $z$  axis with superluminal velocity  $v$ . Namely, let us assume that  $\psi(\boldsymbol{\rho}; t)$ , with  $\boldsymbol{\rho} \equiv (x, y)$ , is a solution of the two-dimensional homogeneous wave equation

$$\left( \partial_x^2 + \partial_y^2 - \frac{1}{c^2} \partial_t^2 \right) \psi(\boldsymbol{\rho}; t) = 0. \quad (1)$$

By applying the *transformation*  $\boldsymbol{\rho} \rightarrow \boldsymbol{\rho} \sin \theta$ ;  $t \rightarrow t - (\cos \theta/c)z$ , the angle  $\theta$  being fixed, with  $0 < \theta < \pi/2$ , one gets [19] that  $\psi(\boldsymbol{\rho} \sin \theta; t - (\cos \theta/c)z) = \psi(x, y, z)$  is a solution to the three-dimensional homogeneous wave equation ( $\nabla^2 \equiv \partial_x^2 + \partial_y^2 + \partial_z^2$ ):

$$\left( \nabla^2 - \frac{1}{c^2} \partial_t^2 \right) \psi \left( \boldsymbol{\rho} \sin \theta; t - \frac{\cos \theta}{c} z \right) = 0. \quad (2)$$

The mentioned theorem holds for the vacuum case, and, in general, is not valid when introducing boundary conditions. However, we discovered that, in the case of a two-dimensional solution  $\psi$  valid on a circular domain of the  $(x, y)$  plane such that  $\psi = 0$  for  $|\boldsymbol{\rho}| = a$ , the above transformation leads us to a (three-dimensional) *localized* solution rigidly traveling with a superluminal speed  $v = c/\cos \theta$  inside a *cylindrical waveguide*; even if the waveguide radius  $r$  will be no longer  $a$ , but  $r = a/\sin \theta > a$ . We can, therefore, obtain an undistorted superluminal solution propagating down cylindrical (metallic) *waveguides* for each (two-dimensional) solution valid on a circular domain. Let us recall that, as well known, any solution to the scalar wave equation corresponds to solutions of the (vectorial) Maxwell equations (cf., e.g., Ref. [8] and references therein).

For simplicity, let us put the origin  $O$  at the center of the circular domain  $\mathcal{C}$ , and choose a two-dimensional solution  $\psi(\boldsymbol{\rho}; t)$  that is axially symmetric, with  $\rho = |\boldsymbol{\rho}|$ , and with the initial conditions  $\psi(\boldsymbol{\rho}; t=0) = \phi(\rho)$  and  $\partial\psi/\partial t = \xi(\rho)$  at  $t = 0$ .

<sup>1</sup>Let us recall that the group velocity is well defined only when the pulse has a clear bump in space; but it can be calculated by the approximate, elementary relation  $v \approx d\omega/dk$  *only* when some extra conditions are satisfied (namely, when  $\omega$  as a function of  $k$  is also clearly bumped).

Notice that, because of the transformations

$$\rho \Rightarrow \rho \sin \theta, \quad (3a)$$

$$t \Rightarrow t - \frac{\cos \theta}{c} z, \quad (3b)$$

the more the initial  $\psi(\boldsymbol{\rho}; t)$  is localized at  $t = 0$ , the more the (three-dimensional) wave  $\psi(\boldsymbol{\rho} \sin \theta; t - (\cos \theta/c)z)$  will be localized around  $z = vt$ . It should be also emphasized that because of transformation (3b) the velocity  $c$  goes into the velocity  $v = c/\cos \theta > c$ .

Let us start with the choice

$$\phi(\rho) = \frac{\delta(\rho)}{\rho}, \quad \xi(\rho) \equiv 0, \quad (4)$$

a formal choice that later on will be relaxed. In cylindrical coordinates the wave equation (1) becomes

$$\left( \frac{1}{\rho} \partial_\rho \rho \partial_\rho - \frac{1}{c^2} \partial_t^2 \right) \psi(\rho; t) = 0, \quad (1')$$

which exhibits the assumed axial symmetry. Looking for factorized solutions of the type  $\psi(\boldsymbol{\rho}; t) = R(\rho)T(t)$ , one gets the equations  $\partial_t^2 T = -\omega^2 T$  and  $(\rho^{-1} \partial_\rho + \partial_\rho^2 + \omega^2/c^2)R = 0$ , where the ‘‘separation constant’’  $\omega$  is a real parameter, which yield the solutions

$$T = A \cos \omega t + B \sin \omega t \quad (5)$$

$$R = C J_0 \left( \frac{\omega}{c} \rho \right),$$

where quantities  $A, B, C$  are real constants, and  $J_0$  is the ordinary zero-order Bessel function [we disregarded the analogous solution  $Y_0(\omega\rho/c)$  since it diverges for  $\rho = 0$ ]. Finally, by imposing the boundary condition  $\psi = 0$  at  $\rho = a$ , one arrives at the base solutions

$$\psi(\rho; t) = J_0 \left( \frac{\lambda_n}{a} \rho \right) (A_n \cos \omega_n t + B_n \sin \omega_n t), \quad \left[ \lambda \equiv \frac{\omega}{c} a \right], \quad (6)$$

the roots of the Bessel function being

$$\lambda_n = \frac{\omega_n a}{c}.$$

The general solution for our two-dimensional problem (with our boundary conditions) will, therefore, be the Fourier-type series

$$\Psi_{2D}(\rho; t) = \sum_{n=1}^{\infty} J_0 \left( \frac{\lambda_n}{a} \rho \right) (A_n \cos \omega_n t + B_n \sin \omega_n t). \quad (7)$$

The initial conditions (4) imply that  $\sum A_n J_0(\lambda_n \rho/a) = \delta(\rho)/\rho$ , and  $\sum B_n J_0(\lambda_n \rho/a) = 0$ , so that all  $B_n$  must vanish, while  $A_n = 2[a^2 J_1^2(\lambda_n)]^{-1}$ ; and eventually one gets

$$\Psi_{2D}(\rho;t) = \sum_{n=1}^{\infty} \left( \frac{2}{a^2 J_1^2(\lambda_n)} \right) J_0 \left( \frac{\lambda_n}{a} \rho \right) \cos \omega_n t, \quad (8)$$

where  $\omega_n = \lambda_n c/a$ .

Let us explicitly notice that we can go on from such a formal solution to more physical ones, just by considering a finite number  $N$  of terms. In fact, each partial expansion will satisfy (besides the boundary condition) the second initial condition  $\partial_t \psi = 0$  for  $t=0$ , while the first initial condition gets the form  $\phi(\rho) = f(\rho)$ , where  $f(\rho)$  will be a (well) localized function, but no longer a  $\delta$ -type function. Actually, the ‘‘localization’’ of  $\phi(\rho)$  increases with increasing  $N$ . We shall come back to this point below.

**IV. LOCALIZED WAVES PROPAGATING SUPERLUMINALLY DOWN (NORMAL-SIZED) WAVEGUIDES**

We have now to apply transformations (3) to solution (8), in order to go on to three-dimensional waves propagating along a cylindrical (metallic) waveguide with radius  $r = a \sin \theta$ . We obtain that Maxwell equations admit in such a case the solutions

$$\Psi_{3D}(\rho, z; t) = \sum_{n=1}^{\infty} \left( \frac{2}{a^2 J_1^2(\lambda_n)} \right) J_0 \left( \frac{\lambda_n}{a} \rho \sin \theta \right) \times \cos \left[ \frac{\lambda_n \cos \theta}{a} \left( z - \frac{c}{\cos \theta} t \right) \right], \quad (9)$$

where  $\omega_n = \lambda_n c/a$ , which are sums over different propagating modes.

Such solutions propagate, down the waveguide, rigidly with superluminal velocity<sup>2</sup>  $v = c/\cos \theta$ . Therefore, (nonevanescent) solutions to Maxwell equations exist, that are waves propagating undistorted along *normal* waveguides with superluminal speed (even if in normal-sized waveguides the dispersion relation for each mode, i.e. for each term of the Fourier-Bessel expansion, is the ordinary ‘‘subluminal’’ one,  $\omega^2/c^2 - k_z^2 = +\Omega^2$ ).

It is interesting that our superluminal solutions travel rigidly down the waveguide: this is at variance with what happens for truncated (superluminal) solutions [7–10], which travel almost rigidly only along their finite ‘‘depth of field’’ and then abruptly decay.

Finally, let us consider a finite number of terms in Eq. (8), at  $t=0$ . We made a few numerical evaluations: let us consider the results for  $N=22$  (however, similar results can be already obtained, e.g., for  $N=10$ ). The first initial condition of Eq. (4), then, is no longer a  $\delta$  function, but results to be the (bumped) two-dimensional wave represented in Fig. 1.

The three-dimensional wave, Eq. (9), corresponding to it, i.e., with the same finite number  $N=22$  of terms, is depicted

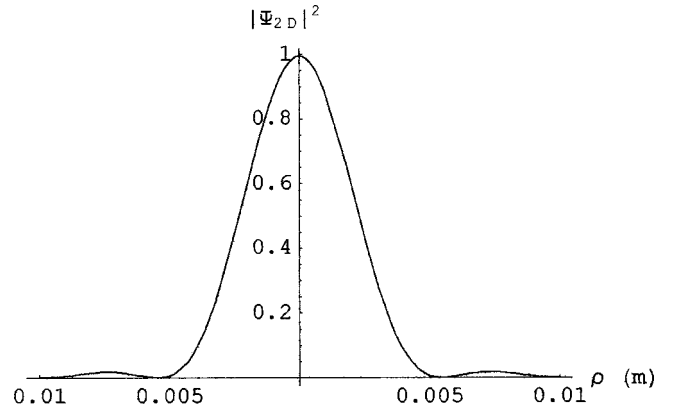


FIG. 1. Shape of the two-dimensional solution of the wave equation valid on the circular domain  $\rho \leq a$ ;  $a=0.1$  m of the  $(x,y)$  plane, for  $t=0$ , corresponding to the sum of  $N=22$  terms in the expansion (8). It is no longer a  $\delta$  function, but it is still very well peaked. By choosing it as the initial condition, instead of the first one of Eqs. (4), one gets the three-dimensional wave depicted in Figs. 2 and 3. The figure height is normalized so that  $|\Psi_{2D}(\rho = 0; t = 0)|^2 = 1$ .

in Fig. 2. It is still an exact solution of the wave equation, for a metallic (normal-sized) waveguide with radius  $r = a/\sin \theta$ , propagating rigidly with superluminal group velocity  $v = c/\cos \theta$ , moreover, it is now a *physical* solution. In Fig. 2 one can see its central portion, while in Fig. 3 it is shown the space profile along  $z$ , for  $t = \text{const}$ , of this propagating wave. It is actually a component of a train of pulses; such pulses being X shaped [8], as it will be better shown elsewhere.

Let us recall that such superluminal motions do not imply causality problems [5,18], since all Bessel beams, and superpositions of them, are known not to allow for superluminal transmission of information [7]. Let us also recall that the fact that our present solutions—when infinitely extended in

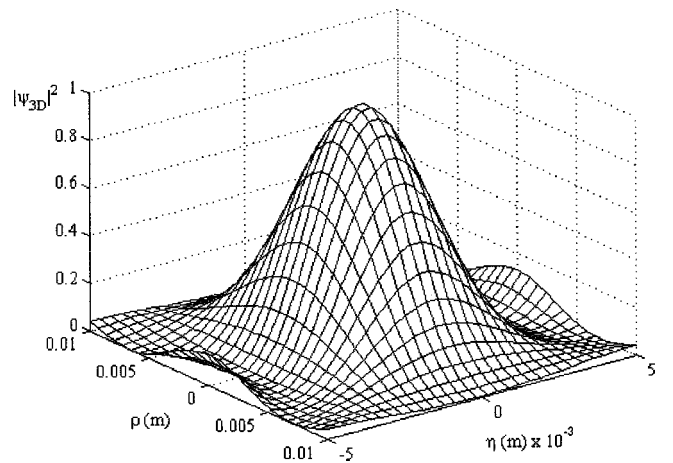


FIG. 2. Three-dimensional (very well localized) wave corresponding to the initial, two-dimensional choice in Fig. 1. It propagates rigidly (along the normal-sized circular waveguide with radius  $r = a/\sin \theta$ ) with superluminal speed  $v = c/\cos \theta$ , as a component of a pulse train. Such pulses are X shaped [8], as it will be better shown elsewhere. Quantity  $\eta$  is defined as  $\eta \equiv [z - (c/\cos \theta)t]$ . The central peak height is normalized so that  $|\Psi_{3D}(\rho = 0; \eta = 0)|^2 = 1$ .

<sup>2</sup>Let us stress that each Eq. (9) represents a *multimodal* (but *localized*) propagation, as if the geometric dispersion compensated for the multimodal dispersion.

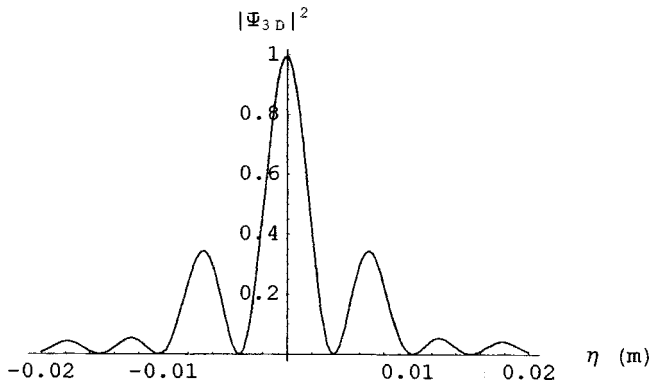


FIG. 3. Shape along  $z$ , for fixed  $t$ , of the three-dimensional wave, in the case when its main peak (shown in Fig. 2) is at the position  $\eta \equiv z - vt = 0$ .

time—possess infinite total energy does not seem to give rise to practical problems (no more than for plane waves), since such kind of pulses can be truncated in time and nevertheless seem to maintain their characteristics of superluminal localized beams along a large field depth [20].

## V. OUR RESULTS FROM THE POINT OF VIEW OF THE STANDARD THEORY OF WAVEGUIDE PROPAGATION

Lu's theorem is certainly a very useful tool to build up localized solutions to Maxwell equations: actually, it can be used to get a variety of solutions, Eq. (9) being just the simplest example. Nevertheless, due to the uniqueness of our previous results, it may be worthwhile to outline an alternative derivation of them that can sound more familiar.

For the sake of simplicity, let us limit ourselves to the domain of TM (transverse magnetic) modes. When a solution in terms of the longitudinal electric component  $E_z$  is sought, one has to deal with the *simple* boundary condition  $E_z = 0$ . We shall look, moreover, for axially symmetric solutions (i.e., solutions independent of the azimuthal variable  $\varphi$ ). (Such choices could be easily generalized, just at the cost of increasing the mathematical complexity.) Quantity  $E_z$  is then completely equivalent to the scalar variable  $\Psi \equiv \Psi_{3D}$  used in the previous analysis.

Let us try to find out solutions of the form

$$E_z(\rho, z; t) = CQ(\rho) \exp\left[i\left(\frac{\omega z \cos \theta}{c} - \omega t\right)\right], \quad (10)$$

where  $Q$  is a function of the radial coordinate  $\rho$  only,  $C$  is a normalization constant, and  $c$  is, here, the velocity of light in the medium filling the cylindrical waveguide, supposing it is nondispersive. The angular frequency  $\omega$  is for the moment arbitrary.

By inserting expression (10) into the Maxwell equation for  $E_z$ , one obtains

$$\rho^2 \frac{d^2 Q(\rho)}{d\rho^2} + \rho \frac{dQ(\rho)}{d\rho} + \rho^2 k_0^2 \sin^2 \theta Q(\rho) = 0, \quad k_0 \equiv \frac{\omega}{c},$$

whose only solution, which is finite on the waveguide axis, is  $Q(\rho) = J_0(\rho k_0 \sin \theta)$ . By imposing the boundary condition

$Q(\rho) = 0$  for  $\rho = r$ , one gets that the acceptable angular frequencies are constrained to be

$$\omega_l = \frac{c \lambda_l}{r \sin \theta}, \quad (11)$$

where  $\lambda_l$  is the  $l$ th zero of the equation  $J_0(x) = 0$ .

Therefore, assuming an arbitrary parameter  $\theta$ , we find that for every mode supported by the waveguide and labeled by the index  $l$ , there is just *one* frequency at which the assumed dependence (10) on  $z$  and  $t$  is physically realizable. Let us show such a solution to be the standard one known from classical electrodynamics. In fact, by inserting the allowable frequencies  $\omega_l$  into the complete expression of the mode, we have

$$E_z^l(\rho, z; t) = C J_0\left(\frac{\rho \lambda_l}{r}\right) \exp\left[i\left(\frac{\omega_l z \cos \theta}{c} - \omega_l t\right)\right]. \quad (12)$$

But the generic solution for (axially symmetric) TM<sub>0l</sub> modes in a cylindrical metallic waveguide is [21]

$$E_z^{\text{TM}_{0l}}(\rho, z; t) = C J_0\left(\frac{\rho \lambda_l}{r}\right) \exp[i(\beta(\omega_l)z - \omega_l t)]$$

with the dispersion relation  $\beta^2(\omega_l) = (\omega_l/c)^2 - (\lambda_l/r)^2$ . By identifying  $\beta(\omega_l) \equiv \omega_l \cos \theta/c$ , as suggested by Eq. (12), and remembering the expression for  $\omega_l$  given by Eq. (11), the ordinary dispersion relationship is obtained. We have, therefore, verified that every term in the expansion (9) is a solution to Maxwell equations not different from the usual one.

The uncommon feature of our solution (9) is that, given a particular value of  $\theta$ , the phase velocity of *all* its terms is always the same, this being independent of the mode index  $l$ ,

$$v_{\text{ph}} = \left[\frac{\beta(\omega_l)}{\omega_l}\right]^{-1} = \frac{c}{\cos \theta}.$$

In such a case it is known that the group velocity of the pulse *equals* the phase velocity [8], which in our example is the velocity *tout court* of the localized pulse.

With reference to Fig. 4, we can easily see that all the allowed values of  $\omega_l$  can be calculated by determining the intersections of the various branches of the dispersion relation with a straight line, whose slope depends on  $\theta$  only. By using suitable combinations of terms, corresponding to different indexes  $l$ , as in our Eq. (9), it is possible to describe a disturbance having a time-varying profile, as already shown in Figs. 1–3 above. The pulse thus displaces itself *rigidly*, with a *velocity*  $v$  equal to  $v_{\text{ph}}$ .

It should be stressed that the velocity  $v$  (or group velocity  $v_g \equiv v$ ) of the pulses corresponding to Eq. (9) is *not* to be evaluated by the ordinary formula  $v_g \approx d\omega/dk$  (valid for quasimonochromatic signals). This is at variance with the common situation in optical and microwave communications, when the signal is usually an “envelope” superimposed onto a carrier wave whose frequency is generally much higher than the signal bandwidth. In *that* case the standard formula for  $v_g$  yields the correct velocity to deal with (e.g., when propagation delays are studied). Our case, on the



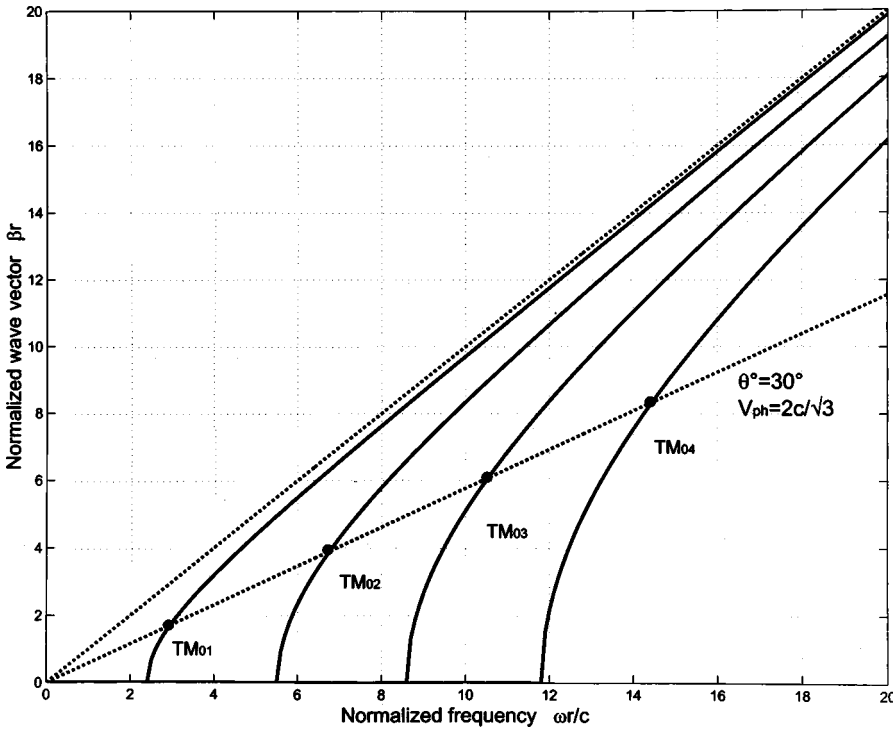


FIG. 4. Dispersion curves for the symmetrical  $TM_{0l}$  modes in a perfect cylindrical waveguide, and location of the frequencies whose corresponding modes have equal phase velocity. [Actually, the phase velocity  $c/\cos \theta$  of all the terms in Eq. (9) is always the same, being independent of the mode index  $l$ . In such a case it is known that the group velocity of the pulse equals the phase velocity: and in our case is the velocity *tout court* of the localized pulse].

contrary, is much more reminiscent of a baseband modulated signal, as those studied in ultrasonics: the very concept of a carrier becomes meaningless here, as the elementary “harmonic” components have widely different frequencies.

Let us finally remark that similar considerations could be extended to all the situations where a waveguide supports several modes. The tests at microwave frequencies should be rather easy to perform; by contrast, experiments in the optical domain would be probably ruled out, at present, by the limited extension of the spectral windows corresponding to not too large attenuations.

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*Note added.* Recently, we came to know that similar work has been done in 1992 by Jian-yu Lu (unpublished).

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